New progress in the stationary D=N=4 supergravity

Oleg V. Kechkin

DEPNI, Institute of Nuclear Physics
M.V. Lomonosov Moscow State University
119899 Moscow, Vorob'yovy Gory, Russia
e-mail: kechkin@depni.npi.msu.su

Abstract

A bosonic sector of the four-dimensional low-energy heterotic string theory with two Abelian gauge fields is considered in the stationary case. A new 4×4 unitary null-curvature matrix representation of the theory is derived and the corresponding formulation based on the use of a new 2×2 Ernst type matrix complex potential is developed. The group of hidden symmetries is described and classified in the matrix-valued quasi General Relativity form. A subgroup of charging symmetries is constructed and representation which transforms linearly under the action of this symmetry subgroup is established. Also the solution generation procedure based on the application of the total charging symmetry subgroup to the stationary Einstein-Maxwell theory is analyzed.

PACS numbers: 04.20.gb, 03.65.Ca

1 Introduction

String theories provide us new physically interesting examples of the field theories which describe some concrete sets of matter fields coupled to gravity [1]. In this paper we consider a bosonic sector of the effective field theory arising in the low-energy limit of heterotic string theory. In [2] it was performed a general toroidal compactification of this theory; in [3] it was shown that in the case of compactification to three dimensions the theory becomes a symmetric space model coupled to gravity on shell. This symmetric space model parameterizes the coset $O(d+1, d+1+n)/O(d+1) \times O(d+1+n)$, where d and n are the numbers of compactified space-time dimensions and original Abelian gauge fields respectively. The corresponding non-linear σ-model allows different null-curvature matrix representations [3], [4]. One of them ([4]) is closely related to the formulation of theory in terms of pair of the real matrix potentials which are the $(d+1) \times (d+1)$ and $(d+1) \times (d+1+n)$ matrix fields. The matrix potential formulation generalizes the conventional complex potential one of the stationary Einstein-Maxwell theory [5] to the heterotic string theory case. This formulation is especially useful for the study of hidden symmetries of the theory; in [6] it was explored for classification of the general low-energy heterotic string theory in the quasi Einstein-Maxwell form and for the determination of the three-dimensional charging symmetry transformations. In [7] it was developed in details a new formalism based on the use of the new $(d+1)\times(d+1+n)$ real matrix potential which undergoes linear transformations when the charging symmetry subgroup acts. This subgroup leaves the trivial values of the all three-dimensional fields unchanged and forms a base for any procedure of the symmetry generation of the asymptotically flat solutions. The most general generation technique of this type is developed in [7] and, in fact, the problem of the most general symmetry extension of the three-dimensional asymptotically flat solutions in heterotic string theory is solved in this work.

However, these general results possess a qualitative modernization in the following sense. For some special values of the parameters d and n one can develop more compact alternative formulations of the theory than the general orthogonal ones which exists for the arbitrary (d, n)-labeled representative of the heterotic string series of the theories. For example, in the case of $d = n = 1$ (the Einstein-Maxwell dilaton-axion theory, see [8], [9]) it is possible to use the compact 4×4 symplectic null-curvature matrix representation instead of the 5×5 orthogonal one. Then, if $d = 2, n = 0$ (the five-dimensional dilaton-Kalb-Ramond gravity), the same situation takes place, but in this case it arises the $SL(4, R)/SO(4)$ matrix representation. In this paper we show, that for the theory with $d = 1, n = 2$, in addition to the conventional orthogonal 6×6 null-curvature matrix, one can use the 4×4 one which parameterizes the coset $SU(2, 2)/S[U(2) \times U(2)]$. We derive the parameterization of this matrix in terms of components of the heterotic string theory fields and relate the new null-curvature matrix to the 2×2 complex matrix potential of the Ernst type. Thus, we show that this theory, which also arises in framework of the $D = N = 4$ supergravity (see [10], [11] for the extremal and black hole solutions), possesses the formal properties of the stationary General Relativity. In particular, we separate the group of the three-dimensional hidden symmetries to the matrix-valued shift, scaling and Ehlers parts (see [12] for comparison with the General Relativity analogies). We also establish another 2×2 complex matrix potential which transforms linearly in respect to the general group of charging symmetry transformations. These new representations derived in this paper for the $d = 1, n = 2$ theory maximally simplify and compactify the results developed for the general low-energy heterotic string theory systems. The new unitary null-curvature matrix based description of the theory seems especially promising for application of the inverse scattering transform method for construction of the soliton solutions [13], [14].

2 A symmetric space model

In the string theory frame the action of the low-energy heterotic string theory reads [1]:

$$S_4 = \int d^4X |\mathrm{det} G_{MN}|^{\frac{1}{2}} e^{-\Phi} \left(R_4 + \Phi_{,M}\Phi^{,M} - \frac{1}{12} H_{MNK}H^{MNK} - \frac{1}{4} F^I_{MN} F^{I\,MN} \right), \tag{1}$$

where $H_{MNK} = \partial_M B_{NK} - \frac{1}{2} A^I_M F^I_{NK} +$ cyclic $\{M, N, K\}$ and $F^I_{MN} = \partial_M A^I_N - \partial_N A^I_M$. Here $M, N, K = 0, ..., 3$, the space-time signature is $(-, +, +, +)$ and $I = 1, 2$. In the stationary case all the fields are t-independent, where $t = X^0$; let us classify the field components in respect to the transformations of the three-dimensional coordinates $x^\mu = X^\mu$. There is the set of the scalars:

$$G = G_{00}, \;\; A = (A^1_0 \; A^2_0), \;\; \phi = \Phi - \log |G|^{\frac{1}{2}}. \tag{2}$$

Then, there are three vectors with the components

$$V_{1\mu} = G^{-1} G_{0\mu}, \; V_{2\mu} = B_{0\mu} + \frac{1}{2} A^I_0 V^I_{3\mu} \;, \; V^I_{3\mu} = -A^I_\mu + A^I_0 V_{1\mu}. \tag{3}$$

Finally, there are two tensor fields

$$h_{\mu\nu} = e^{-2\phi}(G_{\mu\nu} - G V_{1\mu} V_{1\nu}), \quad b_{\mu\nu} = B_{\mu\nu} - \frac{1}{2}(V_{1\mu}V_{2\nu} - V_{1\nu}V_{2\mu}). \tag{4}$$

In fact the field $b_{\mu\nu}$ is non-dynamical; following [3] we put it equal to zero without imposing of any additional restrictions on the other degrees of freedom. The following use of the motion equations allows one to introduce three pseudo scalar fields u, v and s $\quad (s^T = (s^1\ s^2))$ accordingly the relations

$$\begin{aligned}
\nabla \times \vec{V}_1 &= e^{2\phi}(\nabla u + \frac{1}{2}AA^T\nabla v + A\nabla s), \\
\nabla \times \vec{V}_2 &= e^{2\phi}G\nabla v - \frac{1}{2}AA^T\nabla \times \vec{V}_1 + A\nabla \times \vec{V}_3, \\
\nabla \times \vec{V}_3 &= e^{2\phi}(\nabla s + A^T\nabla v) + A^T\nabla \times \vec{V}_1.
\end{aligned} \tag{5}$$

The resulting effective three-dimensional theory can be expressed in terms of the three 2×2 matrices $\mathcal{G}$, $\mathcal{B}$ and $\mathcal{A}$ of the following form [4]:

$$\mathcal{G} = \left(\begin{array}{cc} -e^{-2\phi} + Gv^2 & Gv \\ Gv & G \end{array} \right), \quad \mathcal{B} = \left(\begin{array}{cc} 0 & -(u + \frac{1}{2}As)^T \\ u + \frac{1}{2}As & 0 \end{array} \right), \quad \mathcal{A} = \left(\begin{array}{c} s^T + vA \\ A \end{array} \right). \tag{6}$$

The corresponding three-action reads:

$$S_3 = \int d^3x h^{\frac{1}{2}} \left(-R_3 + L_3\right), \tag{7}$$

where

$$L_3 = \mathrm{Tr}\left[\frac{1}{4}\left(\mathcal{J}_{\mathcal{G}}^2 - \mathcal{J}_{\mathcal{B}}^2\right) + \frac{1}{2}\nabla\mathcal{A}\nabla\mathcal{A}^T\mathcal{G}^{-1}\right] \tag{8}$$

and $\mathcal{J}_{\mathcal{G}} = \nabla\mathcal{G}\,\mathcal{G}^{-1}$, $\mathcal{J}_{\mathcal{B}} = \nabla\mathcal{B}\,\mathcal{G}^{-1}$. To clarify the symmetric properties of the theory it is useful to introduce the following 6×6 real matrix $\mathcal{M}$:

$$\mathcal{M} = \left(\begin{array}{ccc} \mathcal{G}^{-1} & \mathcal{G}^{-1}(\mathcal{B}+\mathcal{T}) & \mathcal{G}^{-1}\mathcal{A} \\ (-\mathcal{B}+\mathcal{T})\mathcal{G}^{-1} & (\mathcal{G}-\mathcal{B}+\mathcal{T})\mathcal{G}^{-1}(\mathcal{G}+\mathcal{B}+\mathcal{T}) & (\mathcal{G}-\mathcal{B}+\mathcal{T})\mathcal{G}^{-1}\mathcal{A} \\ \mathcal{A}^T\mathcal{G}^{-1} & \mathcal{A}^T\mathcal{G}^{-1}(\mathcal{G}+\mathcal{B}+\mathcal{T}) & 1 + \mathcal{A}^T\mathcal{G}^{-1}\mathcal{A} \end{array} \right), \tag{9}$$

where $\mathcal{T} = \frac{1}{2}\mathcal{A}\mathcal{A}^T$. This matrix satisfies the coset relations

$$\mathcal{M}^T = \mathcal{M}, \qquad \mathcal{M}\mathcal{L}\mathcal{M} = \mathcal{M}, \tag{10}$$

where

$$\mathcal{L} = \left(\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{array} \right). \tag{11}$$

The restrictions (10) are preserved by the transformation

$$\mathcal{M} \rightarrow \mathcal{C}^T \mathcal{M} \mathcal{C}, \tag{12}$$

where

$$\mathcal{C}^T \mathcal{L} \mathcal{C} = \mathcal{L}. \tag{13}$$

The Lagrangian L_3, being expressed in terms of $\mathcal{M}$, reads:

$$L_3 = \frac{1}{8} \mathrm{Tr} \left(\mathcal{J}_{\mathcal{M}} \right)^2, \tag{14}$$

where $\mathcal{J}_{\mathcal{M}} = \nabla \mathcal{M} \, \mathcal{M}^{-1}$; it is invariant under the action of the transformation (12)-(13). Thus, this transformation is a symmetry of the theory. In [3] it was originally shown that this map gives a total group of the three-dimensional symmetry transformations.

Our approach is oriented to the close analogy between the heterotic string and Einstein-Maxwell theories. For example, this analogy "generates" the fact that the Lagrangian (8) takes the conventional Einstein-Maxwell form if $\mathcal{G} = f,\ \mathcal{B} = \chi\epsilon,\ \mathcal{A} = V + U\epsilon$, where ϵ is the antisymmetric 2×2 matrix ($\epsilon_{12} = 1$) and the quantities f, χ, U and V are the (nonmatrix) functions. Actually, for the above given choice the motion equations of the theory can be derived from the following effective three-dimensional Lagrangian:

$$L_3 = \mathcal{L}_{EM} = \frac{1}{2f^2} \left| \nabla \mathcal{E} - \bar{\mathcal{F}} \nabla \mathcal{F} \right|^2 - \frac{1}{2} \left| \nabla \mathcal{F} \right|^2, \tag{15}$$

where $\mathcal{E} = f + i\chi - \frac{1}{2}(V^2 + U^2),\ \mathcal{F} = V + iU$. It is exactly the Lagrangian of the stationary Einstein-Maxwell theory; the complex functions $\mathcal{E}$ and $\mathcal{F}$ play the role of the conventional Ernst potentials [5]. Below we show that, moreover, the complete theory under consideration possesses a representation in the pure Einstein theory form. Namely, we derive a formulation of this theory in terms of the single complex 2×2 matrix potential of the Ernst type. This fact leads to the crucial formalism compactification and opens new possibilities in concrete applications of the different powerful methods of solution construction. Actually, from the general theory point of view there are two real 2×2 matrix Ernst potentials $\mathcal{X}$ and $\mathcal{A}$, where $\mathcal{X} = \mathcal{G} + \mathcal{B} + \frac{1}{2}\mathcal{A}\mathcal{A}^T$ [6] which alternatively define the 6×6 orthogonal symmetric space matrix $\mathcal{M}$ (see (9)). However, as it is shown below, it exists the single 2×2 complex Ernst matrix potential E which is uniquely related to some new unitary 4×4 symmetric space matrix M. This new unitary null-curvature matrix representation is actually convenient in framework of application of the inverse scattering transform method [13] to the theory under consideration after its following reduction to two dimensions.

3 New Ernst potential formulation

A general classification of the symmetric space models coupled to gravity in three dimensions is given in [15]. Let us consider the one with the three-dimensional Lagrangian

$$L_c = \frac{1}{4}\left(J_M\right)^2, \tag{16}$$

where $J_M = \nabla M \ M^{-1}$. For us it will be important to restrict the 4×4 null-curvature matrix M by the following unitary coset space relations:

$$M^+ = M, \quad MLM, \tag{17}$$

where

$$L = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{18}$$

The action (16) is invariant in respect to the transformation $M \to C^+ MC$, where $C^+ LC = L$, so we deal with the symmetric space model with the unitary hidden symmetry. Our statement is the following: this new unitary σ-model coincides with the orthogonal one introduced in the previous section. This means that the matrix M can be parameterized by the quantities G, A, ϕ, u, v, and s in such a way that this parameterization will give the general solution of the coset restrictions (17) and also it will guarantee the equality $L_c = L_3$. The rest part of this section is related to determination of the explicit form of this new unitary representation of the theory.

First of all, let us solve the unitary coset restrictions (17); the result reads:

$$M = \begin{pmatrix} P^{-1} & iP^{-1}Q \\ -iQP^{-1} & P + QP^{-1}Q \end{pmatrix}, \tag{19}$$

where $P^+ = P$ and $Q^+ = Q$ are the 2×2 complex matrices. A substitution of Eq. (19) to Eq. (16) shows that

$$L_c = \frac{1}{2}\left(\mathcal{J}_P^2 + \mathcal{J}_Q^2\right), \tag{20}$$

where $\mathcal{J}_P = \nabla P \ P^{-1}$ and $\mathcal{J}_Q = \nabla Q \ P^{-1}$. We plan to calculate the traces in Eqs. (8) and (20) and to comparize the Lagrangians L_3 and L_c. For our purposes it will be convenient to explore the following (temporary) parameterizations of the matrices $\mathcal{G}, \mathcal{B}, \mathcal{A}$ and P, Q:

$$\mathcal{G} = -g_0 \begin{pmatrix} g_1^{-1} + g_1 g_2^2 & g_1 g_2 \\ g_1 g_2 & g_1 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} 0 & -b \\ b & 0 \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} a_1 & a_3 \\ a_4 & a_2 \end{pmatrix},$$
$$P = p_0 \begin{pmatrix} -p_1^{-1} + p_1 |p_2|^2 & p_1 \bar{p}_2 \\ p_1 p_2 & p_1 \end{pmatrix}, \quad P = p_0 \begin{pmatrix} q_0 & q_2 \\ \bar{q}_2 & q_1 \end{pmatrix}. \tag{21}$$

Here the parameters $p_2 = p_2^{'} + i p_2^{''}$ and $q_2 = q_2^{'} + i q_2^{''}$ are complex; all the other ones are real. The parameterization (21) essentially simplifies the comparison procedure. Note, that the signature in P is taken indefinite because the equality $L_3 = L_c$ becomes possible only in this case. After some algebra for the comparing Lagrangians one obtains:

$$L_3 = \frac{1}{2} \Big\{ g_0^{-2} \nabla g_0^2 + g_1^{-2} \nabla g_1^2 + g_1^2 \nabla g_2^2 + g_0^{-2} \Big[\nabla b + \frac{1}{2} \left(a_4 \nabla a_1 - a_1 \nabla a_4 + a_2 \nabla a_3 - a_3 \times \right.$$
$$\left. \times\ \nabla a_2 \right) \Big]^2 - g_0^{-1} g_1 \left[(\nabla a_1 - g_2 \nabla a_4)^2 + (\nabla a_3 - g_2 \nabla a_2)^2 \right] - g_0^{-1} g_1^{-1} \left[\nabla a_2^2 + \nabla a_4^2 \right] \Big\},$$
$$L_c = p_0^{-2} \nabla p_0^2 + p_1^{-2} \nabla p_1^2 + \frac{1}{2} \Big\{ p_0^{-2} p_1^{-2} \nabla q_1^2 + p_0^{-2} p_1^2 \Big[\nabla q_0 - 2 \left(p_2^{'} \nabla q_2^{'} - p_2^{''} \nabla q_2^{''} \right) + \left(p_2^{'2} + \right.$$
$$\left. + p_2^{''2} \right) \nabla q_1 \Big]^2 \Big\} - p_0^{-2} \left[\left(\nabla q_2^{'} - p_2^{'} \nabla q_1 \right)^2 + \left(\nabla q_2^{''} + p_2^{''} \nabla q_1 \right)^2 \right] - p_1^2 \left[\nabla p_2^{'2} + \nabla p_2^{''2} \right]. \tag{22}$$

Now the identification process can be performed in the following way. From comparison of the coefficients before the brackets in the last four ‘negative’ terms one concludes that $g_0^{-1} g_1^{-1} \sim p_1^2$ and $g_0^{-1} g_1 \sim p_0^{-2}$. In this case two first ‘positive’ terms are equal with the arbitrary choice of the coefficients of proportionality. Let us put

$$g_0 = p_0 p_1^{-1}, \; g_1 = p_0^{-1} p_1^{-1}, \; g_2 = q_1; \tag{23}$$

then three first terms coincide evidently. It is easy to see that if

$$a_1 = \sqrt{2} \left(q_2^{'} - p_2^{'} q_1 \right), \; a_2 = \sqrt{2} p_2^{''},$$
$$a_3 = \sqrt{2} \left(q_2^{''} + p_2^{''} q_1 \right), \; a_4 = -\sqrt{2} p_2^{'}, \tag{24}$$

then the four last terms also become equal. The last step is to substitute Eqs. (23) and (24) to fourth term of L_3 in Eq. (22). This allows one to compare the remaining non-identified parameters b and q_0. The result reads:

$$b = q_0 - p_2^{'} q_2^{'} + p_2^{''} q_2^{''}. \tag{25}$$

Thus, Eqs. (23)-(25) give the solution of the our problem. Using them and Eqs. (6) and (21) one finally has:

$$P=\frac{e^{-\phi}}{|G|^{\frac{1}{2}}}\left(\begin{array}{cc} -G-\frac{1}{2}[(A_0^1)^2+(A_0^2)^2] & \frac{1}{\sqrt{2}}(A_0^1+iA_0^2) \\ \frac{1}{\sqrt{2}}(A_0^1-iA_0^2) & -1 \end{array}\right), \quad Q=\left(\begin{array}{cc} u & \frac{1}{\sqrt{2}}(s^1+is^2) \\ \frac{1}{\sqrt{2}}(s^1-is^2) & v \end{array}\right). \quad (26)$$

It is interesting to note, that the matrix P is constructed from the scalar field components, whereas Q is defined by the pseudo scalar ones only. This opportunity essentially simplifies the translation of the solution obtained in the σ-model terms to the language of the heterotic string theory field components.

4 Study of hidden symmetries

Now let us introduce the potential

$$E = P + iQ; \quad (27)$$

then the effective three-dimensional matter Lagrangian $L_c \equiv L_3$ takes the following form:

$$L_3 = 2\mathrm{Tr}\left[\nabla E\left(E+E^+\right)^{-1}\nabla E^+\left(E+E^+\right)^{-1}\right]. \quad (28)$$

It is easy to see that the scale transformation

$$E \rightarrow S^+ES \quad (29)$$

is a symmetry for the arbitrary non-degenerated constant matrix S. Then, for the arbitrary Hermittean constant matrix R the shift

$$E \rightarrow E + iR \quad (30)$$

is also a symmetry. To establish the rest part of the symmetry transformations let us note that the map

$$E \rightarrow E^{-1} \quad (31)$$

is a discrete symmetry of L_3. This map transforms the scale transformation (29) to itself (with the re-parameterization $S \rightarrow (S^+)^{-1}$), whereas from the shift transformation (30) one obtains the following new symmetry:

$$E^{-1} \rightarrow E^{-1} + iL, \quad (32)$$

where $L^+ = L$. From Eqs. (30) and (32) it follows that the map (31) relates the shift and this new nonlinear symmetry by the replacement $R \leftrightarrow L$.

Eqs. (29), (30) and (32) give the total group of the hidden symmetry transformations of the theory under consideration. In fact this group coincides with the unitary group discussed in the previous section (in Eq. (29) the common phase multiplier in S is not sufficient). The transformations (29), (31) and (31) are the straightforward matrix generalizations of the scale, shift and Ehlers symmetries of the stationary General Relativity (see [16], [17] and [12]). In this remarkable analogy the quantity E plays the role of the matrix Ernst potential.

From Eqs. (26) and (27) it follows that the trivial value of the all scalar and pseudo scalar three-fields leads to the special value $E_0 = \sigma_3$ of the matrix Ernst potential E. Let us establish the explicit form of subgroup of the three-dimensional charging symmetries, i.e., the form of the symmetry transformations which preserve the above defined three-dimensional trivial field configuration. To do this, let us introduce the following new matrix potential Z:

$$Z = 2\left(E + \sigma_3\right)^{-1} - \sigma_3. \tag{33}$$

Then after some algebra from Eq. (28) one obtains that

$$L_3 = 2\mathrm{Tr}\left[\nabla Z\left(\sigma_3 - Z^+\sigma_3 Z\right)^{-1}\nabla Z^+\left(\sigma_3 - Z\sigma_3 Z^+\right)^{-1}\right]. \tag{34}$$

It is easy to see that the transformation

$$Z \to e^{i\alpha} C_L Z C_R \tag{35}$$

describes a symmetry of Eq. (34) if the parameter α is real and both the C_L and C_R matrices satisfy the $SU(1,1)$ group restrictions:

$$C_L^+\sigma_3 C_L = C_R^+\sigma_3 C_R = \sigma_3, \quad \det C_L = \det C_R = 1. \tag{36}$$

From comparison with the orthogonal group realization of the charging symmetry subgroup of transformations given in [6] it immediately follows that Eqs. (35) and (36) give the general charging symmetry subgroup of the theory.

Now let us briefly discuss the possible application of the established unitary formulation to the problem of generation of the asymptotically flat solutions. Such a generation is based on the use of charging symmetries, because the asymptotically flat solutions are trivial at the spatial infinity and only the subgroup of charging symmetry transformations preserve this property. It is easy to see that for the trivial solution $Z = Z_0 = 0$, and this special value is actually preserved by the linear and homogenios transformation (35). It is possible to start our generation from the subsystem with

$$Z = \begin{pmatrix} z \\ 0 \end{pmatrix}, \tag{37}$$

where $z = (z_1 \; z_2)$. It is easy to verify that Eq. (37) actually defines a subsystem, i.e. that the anzats under consideration is actually consistent. One can prove that the corresponding motion equations can be obtained from the effective Lagrangian

$$L_{EM} = 2 \, \frac{\nabla z \left(\sigma_3 - z^+ z\right)^{-1} \nabla z^+}{1 - z \sigma_3 z^+}, \tag{38}$$

which exactly coincides with the one given by Eq. (15). Actually, the substitution (see also [18])

$$z_1 = \frac{1 - E}{1 + E}, \qquad z_2 = \frac{\sqrt{2} F}{1 + E} \tag{39}$$

gives a proof of this statement immediately. Thus, the subsystem (37) coincides with the stationary Einstein-Maxwell theory and one can start from the asymptotically flat Einstein-Maxwell fields for generation of the heterotic string theory solutions accordingly Eq. (35). In this generation one can omit the transformation subgroup given by the phase factor $e^{i\alpha}$ and the group matrix C_R if one starts from the charging symmetry complete Einstein-Maxwell fields. Actually, it is easy to see that the map

$$z \rightarrow e^{i\alpha} z C_R \tag{40}$$

is a symmetry of Eq. (38) which preserve the trivial z-value. In fact it is exactly the subgroup $U(1) \times SU(1,1)$ of the Einstein-Maxwell theory charging symmetries [12]. Thus, in this sense, the most general procedure of the continuous symmetry extension of the asymptotically flat Einstein-Maxwell solutions to the field of heterotic string theory under consideration is given by Eqs. (35) and (37) with arbitrary $C_L \in SU(1,1)$ and $C_R = 1$, $\alpha = 0$. We hope to present explicit generation examples in the forthcoming publications. At the end of this section let us note that the formulation of the theory based on the use of the complex matrix potential Z allows one to obtain results in the charging symmetry invariant form. This statement is related to the both symmetry technique application and straightforward solution construction. In the latter case one must use an anzats which is given by restrictions invariant in respect to the charging symmetry transformation (35).

Acknowledgments

This work was supported by RFBR grant N^0 00 02 17135.

5 Conclusion

The main results of this paper are the E- and Z-based matrix potential formalisms of the theory. We have found an explicit form of the parameterization of the Ernst matrix potential E in terms of the heterotic string theory physical field components. Also we have studied the group of hidden symmetries of the theory in framework of these new approaches. In this study the matrix potential Z is especially convenient for the work with asymptotically flat solutions. We have also indicated a possibility of generation of the heterotic string theory solutions starting from the stationary Einstein-Maxwell fields. The results obtained in this paper concludes the investigation performed in [19]. The $\mathcal{Z}$-formalism developed in this paper is explicitly related to the unitary null-curvature matrix formulation and seems really promising for the following application of the inverse scattering transform method to this theory. This statement is more than only a hope in view of the role which the conventional Ernst potential formulation play in construction of the soliton solution in the classical General Relativity. Now the use of the matrix Ernst potential E guarantees the positive result for this activity in the field of the discussed four-dimensional heterotic string theory. This result is controlled by the close analogy between the Einstein and heterotic string theories in three and lower space-time dimensions. The generalization of the obtained results to the case of the heterotic string theory with the arbitrary numbers of toroidally compactified dimensions and Abelian gauge fields can be performed in framework of the special projective formalism developed in [20]. This generalization will also possess the charging symmetry completeness property for the generated asymptotically flat solutions.

References

[1] E. Kiritsis, "Introduction to superstring theory", Leuven Univ. Pr. (1998).

[2] J. Maharana, J.H. Schwarz, Nucl. Phys. **B390** (1993) 3.

[3] A. Sen, Nucl. Phys. **B434** (1995) 179.

[4] A. Herrera-Aguilar, O.V. Kechkin, Int. J. Mod. Phys. **A13** (1998) 393.

[5] F. J. Ernst, Phys. Rev. **167** 5 (1968) 1175.

[6] A. Herrera-Aguilar, O.V. Kechkin, Phys. Rev. **D59** (1999) 124006.

[7] O.V. Kechkin, “3D heterotic string theory: new formalism and extremal solutions”, submitted to Nucl Phys. **B**.

[8] D.V. Galtsov, O.V. Kechkin, Phys. Rev. **D54** (1996) 1656.

[9] D.V. Galtsov, O.V. Kechkin, Phys. Lett. **B361** (1995);

[10] E. Bergshoeff, R. Kallosh, T. Ortin, Nucl. Phys. **B478** (1996) 156.

[11] K. Behrndt, D. Lust, W.A. Sabra, Nucl. Phys. **B510** (1998) 264.

[12] D. Kramer, H. Stephani, M. MacCallum, E. Herlt, “Exact solutions of the Einstein field equations”, Deutcher Verlag der Wissenschaften, Berlin, (1980).

[13] V.A. Belinsky, V.E. Zakharov, JETP **48** (1978) 985; JETP **50** (1979) 1.

[14] A. Eris, A. Karasu, M. Gurses, J. Math. Phys. **25** (1984) 25.

[15] P. Breitenlohner, D. Maison, G.W. Gibbons, Commun. Math. Phys. **120** (1988) 295.

[16] J. Ehlers, in “Les Theories de la Gravitation” (1959) CNRS, Paris.

[17] W. Kinnersley, J. Math. Phys. **18** (1977) 1529.

[18] P.O. Mazur, Acta Phys. Pol. **B14** (1983) 219.

[19] O.V. Kechkin, “Generation of heterotic string theory solutions from the stationary Einstein-Maxwell fields”, accepted for publication in Phys. Lett. **B**.

[20] O.V. Kechkin, “Heterotic string theory interrelations”, submitted to Phys. Rev. Lett.